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REPORT No. 889

On the Computational Procedures for Firing and Bombing Tables

S. GORN

M. L. JUNCOSA

DEPARTMENT OF THE ARMY PROJECT No. 503-06-002
ORDNANCE RESEARCH AND DEVELOPMENT PROJECT No. TB3-007K

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

ERRATA SHEET

for

BRL REPORT NO. 889

ON THE COMPUTATIONAL PROCEDURES FOR FIRING AND BOMBING TABLES

by

S. Gorn
M.L. Juncosa

Page 9 - 14 lines from the bottom -

for "and the mame-ographed"

read "and the mime-ographed".

Page 11 - line 7

for "data-gathering techniques if referred".

read "data-gathering techniques is referred".

Page 12 - 2 lines from bottom -

for "and $\overline{x}$ us the"

read "and $\overline{x}$ is the"

Page 17 - line 4

for "$W.D. = \frac{1.01}{x_\omega} \quad w_z \left(t_\omega - \frac{x_\omega}{x_o}\right)$"

read "$W.D. = \frac{1.01}{x_\omega} \quad w_z \left(t_\omega - \frac{x_\omega}{\dot{x}_o}\right)$"

Page 18 - line 8 -

for "$F = \frac{4c(P.E.)}{100} \quad x_\omega$"

read "$F = \frac{4c(P.E.)\, x_\omega}{100}$"

Page 21 - line 19 from bottom -

for "and for higher altitude drops"

read "and for higher values of M using data from trajectories of high Altitude drops".

SUBJECT: Errata in BRL Report No. 889

Page 24 - line 2

for "1. $t_{\omega}' = t_{\omega} - k_{q}$, (secs.)"

read "1. $t_{\omega}' = t_{\omega} - kq$, (secs.)".

Page 24 - line 5 -

for "4. $\mu = 1000/y^*$ (mils)"

read "4. $\mu' = \frac{1000 \lambda '}{y^*}$ (mils)"

Page 31 - line 7 -

for "Roy. Soc. Lone."

read "Roy. Soc. Lond."

Page 31 - line 4 from bottom -

for "BRL Report (1934)."

read "BRL Report No. X-102 (1934)".

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

REPORT NO. 889

JANUARY 1954

ON THE COMPUTATIONAL PROCEDURES FOR FIRING AND BOMBING TABLES

S. Gorn

M. L. Juncosa

Department of the Army Project No. 503-06-002
Ordnance Research and Development Project No. TB3-007K

A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

REPORT NO. 889

SGorn/MLJuncosa/ekb
Aberdeen Proving Ground, Md.
January 1954

ON THE COMPUTATIONAL PROCEDURES FOR FIRING AND BOMBING TABLES

ABSTRACT

This report describes the methods and means of computing ground-to-ground firing tables and air-to-ground bombing tables at the Computing Laboratory, BRL. The description includes all steps in the transformation from measured data into printed tables. It includes a detailed discussion of the effect of the use of high speed digital computers in various portions of the computations, and discusses further the expected effect of further "mechanization".

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INTRODUCTION

This report is primarily concerned with the description of the computational procedures used at present at the Aberdeen Proving Ground to produce firing and bombing tables. The data gathering methods and the field requirements on the tables will be mentioned only insofar as they affect these procedures.

These present procedures were developed continuously over a number of decades. At the beginning of the first world war, except for the case of mortar fire, the only trajectories being computed in this country were for direct, low angle fire, and the Siacci methods were used (see Ingalls' Tables - Artillery Circular M). With the introduction of anti-aircraft guns, it became apparent that this was insufficient. At Sandy Hook and at Aberdeen, Lt. Philip Alger began the computation of AA trajectories by the French Short Arc methods in 1917. Early in 1918, Capt. F. W. Loomis, who had visited the French and British ballisticians, introduced at Aberdeen, Fowler's method of numerical integration. From then on the contributions of well known mathematicians to the development of the present conputational methods were continual, beginning with Major Veblen's Range Firing and Computing Section at Aberdeen, and followed shortly thereafter by Major Moulton's group at the Office of Chief of Ordnance in Washington. Veblen's group included Bliss, Wiener, Gronwall, Franklin, Gill, Widder, and H. H. Mitchell. Moulton's group included Ritt, A. A. Bennett, Milne, D. Jackson, Ford, Alexander, Buck, Walsh, Vandiver, Dresden, and Dantzig. In the past decade, further development of these procedures, including the development of bombing table computations, paralleled the growth and emergence of the Computing Laboratory as a separate unit under A. A. Bennett, T. E. Sterne, L. S. Dederick, and W. W. Leutert. However, since the developments and refinements in the overall procedures appeared at different times, until now no complete picture has been given. To give such a picture of the procedures used to obtain two fairly typical kinds of tables is one of the purposes of this report.

This, of course, does not mean that developments and refinements in these procedures will not continue. On the contrary, the pressure of the development of new weapons, the discovery and improvement of measuring instruments and techniques, the advancement of the physical theories, the evolution of new mathematical methods, and the construction and procurement of more powerful computing devices will force these computational procedures to change. Indeed, another purpose of this report is to serve as a reference for those who are in a position to encourage and implement such future improvements of the procedures as result from the above.

In addition, this report serves as a more detailed description of the execution of an essential mission of the Computing Laboratory and acquaints new personnel with the details of this mission.

From the statement of the above three purposes for this report it is clear that it is not intended to establish or advocate a standard operating procedure for the production of Ballistic tables; nor is it intended to recommend perpetuation of present practices.

By the "computational procedures" we have been talking about we mean the complete programs for transforming the measured data into tables; these programs involve the use of many sub-procedures including hand computations and various computations by possibly the same or different high speed digital computing machines. Very many of the overall procedures have been recently mechanized, and more mechanization is certainly possible. A further purpose of this report is to point out where further mechanization is possible. However, let us remark here that the chain of a completely mechanized program should be broken at certain critical points, if only to examine for the effects of bad data which may need rejecting. With machines having available much larger memories than the present ones nevertheless one can see that even this examination for bad data could be mechanized.

The computational procedures we will describe are those of the classical problems of ground to ground firing tables where the shell's trajectories are not tracked on the range and tables for bombing of fixed targets from aircraft where the bomb's trajectories are tracked. There are, of course, other ballistic tables; but as will be pointed out in the concluding remarks, many of the features in their production appear in the production of the above two types. The terms firing and bombing tables without further qualification will in this report always refer to these two types.

Let it suffice for the moment to say that when the block flow charts of the computational procedures for firing and bombing tables (see figures 1 and 2) are compared, as is to be expected, striking similarities in structure are apparent. There are also striking differences, one of the most readily observable ones being the ability to adjust the drag coefficient for each bombing table because the missile is tracked, while for firing tables one can only assume a drag coefficient and adjust the ballistic coefficient because at present the missile is not tracked. This explains why in the reductions at present the bombing tables branch requires more personnel than the firing tables branch. The use of accelerometers in range-dropped bombs to determine the drag on the bomb and the possible use of Doppler techniques to track gunfire may make the work of reduction per missile about the same for both branches. However, it should be remarked that, since the present firing tables are more than accurate enough in view of varying meteorological conditions and the lack of sharp control over the muzzle velocities, tracking gunfire missiles is an unnecessary expense.

Acknowledgement of the authors' gratitude are due to J. Prevas, H. Reed and others of the Firing Tables Branch, and to E. H. Martin, M. Field and others of the Bombing Tables Branch for extensive time and pertinent details. The authors also thank L. Butler, E. Gersten and others of the Coding Sections for details of the programs and their running times. They are also grateful to A. A. Bennett and T. E. Sterne for their critical reviews and advice.

II - THE TRAJECTORY EQUATIONS

The physical assumptions behind the present computational procedures producing firing and bombing tables are that the shell or bomb are essentially particles. For the tables discussed in this report the only concession in the equations used in the computation to the fact that the projectile is not really a particle is that there is a drag force. This force per unit mass is given by a resistance function one of whose factors is a drag coefficient. Except in air to air trajectories [6]*, no computationally feasible theory for spin-and fin-stablized shell or bombs with a general yaw is in use now. Small yaw theories have existed for some time [3], [5], [8]. But because of their computational difficulties the effects of yaw on the trajectory

*Such numbers refer to the bibliography

have only been computed in some special cases. (We remark that at present studies to incorporate the effects of yaw in more general cases are continuing [10]).

The equations for a particle trajectory with drag and including (see remarks on omissions below) the effects of wind and the Coriolis force due to the earth's rotation are

$$
\begin{aligned}
\ddot{x} &= -E\,(\dot{x} - w_x) + \lambda_1\,\dot{y},\\
\ddot{y} &= -E\,\dot{y} - g - \lambda_1\,\dot{x}, \qquad (1)\\
\ddot{z} &= -E\,(\dot{z} - w_z) + \lambda_3\dot{y} + \lambda_2\dot{x}
\end{aligned}
$$

where x is distance down range, y is vertical distance, z is horizontal distance to the right (looking down range), $(\lambda_3, -\lambda_2, -\lambda_1)$ are twice the values of the components of the earth's angular velocity in the (x, y, z) system, E is the resistance function, and g is the acceleration due to gravity. The vertical wind component, and the Coriolis contributions involving $\dot{z}$ are omitted, being very small compared to the remaining terms in the equations (1). (In the bombing reductions the range grid coordinates, which differ only by a slight rotation from the above, are used.)

For firing tables g is treated as a constant except for trajectories with very high summits (of the order of five miles). For bombing tables

$$
g = g_o\,(1 - \frac{2y}{r}) \qquad (2)
$$

where g_o is a constant and r is the earth's radius. The resistance function E is given by

$$
E = \frac{\rho(y)\ u\ K_D(M)}{C} \qquad (3)
$$

where $\rho(y)$ is the air density at the height y, u is the projectile's speed relative to the air, $K_D(M)$ is the drag coefficient, an empirical function of the Mach number, M ($=u/a$), of the projectile, and C is the ballistic coefficient which is given by

$$
C = \frac{m}{id^2} \qquad (4)
$$

where m is the mass of the projectile, d is its caliber and i is the form factor (a constant near unity).

For bombing tables it is customary to write

$$E = \gamma\, \rho(y)\, u\, K_D \tag{5}$$

where γ is the reciprocal of the ballistic coefficient. (Sometimes $\rho(y)$ is replaced by $H(y)$, the ratio of the air density at the height y to that at sea level and then γ is redefined as the quotient of the density at sea level by the ballistic coefficient. This is done to replace handling the large number of the very small values obtained for the density, $\rho(y)$, by handling the same number of moderate values obtained for the relative density, $H(y)$.)

For firing tables two forms of E are used. In the tradition of the work of the Gavre Commission in the late nineteenth century and with later modifications to include the dependence of the drag on the speed of sound E has been written as

$$E = \frac{(a/a_s)\, \rho(y)\, G(u^2/a^2)}{C} \tag{6}$$

where a/a_s is the ratio of the speed of sound a at the height y to that a_s at sea level and G is called the drag function, an empirical function originally considered to depend only on u and now tabulated as a function of $(u/a_s)^2$. When used in computation u^2 is multiplied by $(a_s/a)^2$ and this product used as the independent variable for entry into the G - table, the form more in line with the current aerodynamic usage is that of (3).

The trajectory equations (1) are solved numerically for x, y, and z by the modified Euler method of numerical integration with one iteration (sometimes called the Heun method). The general procedure is as follows. First the system of equation to be integrated is written as a system of first order equations of the type

$$\mathring{V}(t) = F\,(V, t) \tag{7}$$

where V and F are vectors. Then a first approximation to $V(t+\Delta t)$ is given by

$$\overline{V}\,(t+\Delta t) = V(t) + F\,(V,\, t)\Delta t. \tag{8}$$

the next and final approximation is given by

$$V(t+\Delta t) = V(t) + \left[F(V,t) + F(\overline{V},\, t+\ \ t)\right]\Delta t/2. \tag{9}$$

In the case of the so-called reduction trajectories, described in section III, V and F have six components, namely

$$
\begin{array}{ll}
V_1 = x & f_1 = V_2 \\
V_2 = \overset{\circ}{x} & f_2 = -E\left[V_2 - w_x(V_3)\right] + \lambda_1 V_4 \\
V_3 = y & f_3 = V_4 \\
V_4 = \overset{\circ}{y} & f_4 = -EV_4 - g - \lambda_1 V_2 \\
V_5 = z & f_5 = V_6 \\
V_6 = \overset{\circ}{z} & f_6 = E\left[V_6 - w_z(V_3)\right] + \lambda_3 V_4 + \lambda_2 V_2
\end{array}
$$

In the case of the so-called normal trajectories, V and F have four components, the last two components being deleted as well as the terms in the f's which involve the w's and the λ's. In each case Δt is varied with the speed. This is necessary because the usually encountered K_D vs. M curve is slowly varying up to about M = .9 or so; then it rapidly takes on a relatively large positive curvature and then rises sharply somewhere between M = .96 or so and M = 1.2 or so; then the curve takes on a large negative curvature and finally settles down as a slowly varying function again. The range of the argument of K_D is divided into five speed intervals characterized by the above behavior. At each step in the integration a discrimination is performed to determine in which of these five intervals the speed of the projectile lies. For all the steps for which the projectile is in a particular speed interval Δt is a constant. These five constant values of Δt have been determined by experience with many K_D's.

We add here that very extensive discussions of the theory and of the hand computation of firing tables are given in the books by Moulton [7], Bliss [1] Hayes [4], Kelley, McShane and Reno [6] and the memeographed notes by Dederick [2], and the report by Hitchcock [13].

III - FIRING TABLES

A. General Remarks

Firing tables, generally speaking, are tables of data for use in aiming and timing bullets, shell, rockets, or other projectiles. We will limit most of our discussion of firing tables to tables used in ground gun fire. In addition to range versus elevation, drift, and time of flight, among the most useful quantities in such tables are probable errors in range and deflection, range effects of increase in muzzle velocity and in rear wind, change in range for 1 mil change in elevation, and the like. These data are needed by artillerymen, manufacturers of munitions and of aiming equipment, and others. These tables must be produced by

a. executing a series of firings and making relevant measurements including measurements of the various non-standard conditions at firing time,

b. computing a series of pertinent trajectories to estimate the ballistic coefficient C and the effects of variation in C (These are the so-called "reduction trajectories"),

c. computing a series of trajectories under "normal" conditions, i.e., with $w_x = w_z = \lambda_1 = \lambda_2 = \lambda_3 = 0$ and a standard atmosphere for $H(y)$ (these are the so-called "normal trajectories"), and then the effects of variation in weight, muzzle velocity, atmospheric conditions, and angle of elevation on the range, and

d. computing a correction formula for time of flight and a formula for drift, and finding probable errors in range.

A block flow chart describing these steps in further detail may be found in Fig. 1.

Let us now consider what is done in the main blocks of this flow chart, and how the results are obtained.

B. Collection of Range Data:

At the firing range the position of the trunnion of the gun is measured by a transit. The azimuth and elevation of the gun are set and measured using a transit for the azimuth and a muzzle clinometer for elevation. The projectile is then weighed, magnetized, and placed in the gun. The time of firing is recorded. The initial speed is determined by the use of a chronograph and two parallel coils suspended ahead of the muzzle of the gun in such a manner that the axes of the coils and the muzzle of the gun are in line. (This last set up is called a "cage"). The magnetized projectile induces currents in each of these coils at slightly different times which are measured by the chronograph; this time difference together with the distance between the coils is sufficient to determine the average speed through the cage. Since because of blast the true speed cannot be measured exactly at the muzzle, an effective muzzle speed is obtained by an extrapolation from the average speed through the cage. The impact point which is usually on water, is observed from three or four towers using a theodolite, and the time of flight is determined by stop watch. (Whenever the projectile is fuzed to burst in the air, the time of flight to the burst point is determined by a chronograph.) These data are gathered for various muzzle speeds and initial angles of elevation, there usually being approximately ten rounds per group with five or six different elevations for each muzzle speed and from one to eight different muzzle speeds.

While the firings are going on meteorological data (often briefly called metro data) are gathered hourly from radiosondes in ascending balloons which are tracked by direction finders and theodolites.

These data include range wind w_x, cross wind w_z, air temperature T, and humidity, all as functions of altitude y. The height of tides, which is necessary to get the height of impact, (most of firings are over water) is obtained at the same time from a gauge stick. (Since the purpose of this report is to present the logical aspects of the computational procedure, the reader interested in further details of the data-gathering techniques if referred to the many pertinent BRL reports and such comprehensive books as that of Cranz [11] .)

These raw data must then be processed by preliminary reductions to determine the parameters, functional as well as numerical, which enter into the reduction trajectories, for the computation of the ballistic coefficient, C.

C. Preliminary Reductions:

The numerical parameters are the mass of the projectile and initial conditions required for solving the trajectories (1), namely, muzzle coordinates, x_o, y_o, z_o, muzzle speed, V_o, elevation, ϕ, and muzzle velocity components, $\dot{x}_o$, $\dot{y}_o$, which except for ϕ are obtained by simple calculations on the raw data. However, the ϕ obtained from the muzzle clinometer measurements is corrected by comparing ranges of a few computed low angle trajectories with observed ranges. (This correction is called jump; it covers the result of many effects of the vertical motion of the gun during firing.)

The functional parameters required are the drag coefficient $K_D(M)$, or the drag function G, the air density $\rho(y)$, the speed of sound $a(y)$, the absolute temperature $T(y)$, and the wind components $w_x(y)$ and $w_z(y)$. The temperature $T(y)$ is used to obtain $a(y)$ through the relation

$$\frac{a}{a_s} = \sqrt{\frac{T}{T_s}} \tag{7}$$

where the subscript s denotes that the quantity modified by the subscript is evaluated at sea level under standard conditions. It is also used with the humidity and pressure to obtain the density $\rho(y)$.

Since the reduction trajectories are to be trajectories representative of a group of firings under similar conditions, those numerical and functional parameters which vary from shot to shot are averaged. The various averagings are performed as follows.

The mean muzzle velocity, v_o, for a group is computed as the arithmetic mean of the observed values for all the firings of the group. Since the elevations are reset after each firing, and the jump adjustment is uniform, this implies that the muzzle velocitity components for $\mathbf{v}_o$ are also the arithmetic means of the components of the observed muzzle velocities, corrected by jump.

The impact points for the firings at the Aberdeen Proving Ground are usually on water. The height of tides for the group is taken as the height at the mean time of fire. This is used to determine the mean height of impact for the group.

The terminal values of the time of flight, t_ω, the range, x_ω, and the deflection, z_ω, as well as the mass of the projectile for our average group trajectory are also determined by taking arithmetic means. The group time of flight, t_ω, are used to find corrections for the times of flight to be entered in the tables. The group ranges, x_ω, are used in the criteria establishing the form factor, i, during the running of the reduction trajectories. They, as well as the group deflections, z_ω, and the sampling statistics to be described shortly are used in the computation of probable errors in range and deflection.

The averaging procedure for the metro data (namely T, w_x, w_z, and ρ) is slightly more complicated. First the mean times of firing for those portions of the group of rounds between successive metro ascensions is found; then the metro data are interpolated for equal intervals in altitude at each ascension; then a further interpolation is made to the mean times for these equal intervals in altitude; and finally the metro data at equal y for the various mean times are weighted by the number of rounds between the various successive ascensions to obtain the mean metro data for the whole group.

With the completion of all these reductions of the raw data, which are, incidentally, being done by hand, a drag coefficient or drag function, K_D or G is chosen from among many available. This choice is based upon experience in the past with projectiles of all shapes and sizes.

The final reduction made before a first estimate of the ballistic coefficient C is made and the reduction trajectories run is the analysis of the observed probable errors of range and deflection made to minimize possible trends in wind and velocity. This is done as follows. First the usual sampling estimate of the variance is obtained:

$$A_o = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

where $x_1, \ldots, x_n$ are the quantities (ranges or deflections) in the group in order of firing, of which there are about 10, and $\bar{x}$ us the arithmetic mean of these quantities. Then the quantity

$$A_1 = \frac{\sum_{i=1}^{n-1} (x_i - x_{i+1})^2}{2(n-1)}$$

is computed, and $0.6745 \min(\sqrt{A_0}, \sqrt{A_1})$ is accepted as being sufficiently close to the probable error with the effect of the trends in the wind and velocity removed.* Sometimes the above quantities are multiplied by a Beta function to correct for sample size.

Footnote: *A basis for this choice is the following: if the firings are assumed to be independent, and the single shot expectations for x_i are a_i, and all the shots are assumed to have the same variance, σ^2, then it is not difficult to show that the expected value of A_0 is

$$\sigma^2 + \frac{\sum_{i=1}^{n} a_i^2 - \frac{1}{n}\left[\sum_{i=1}^{n} a_i\right]^2}{n-1},$$ and that of

A_1 is $\sigma^2 + \dfrac{\sum_{i=1}^{n-1} (a_i - a_{i+1})^2}{2(n-1)}$; in particular, for a linear trend with

firings at equal time intervals, $a_i = ai + b$, we would have that the expected value of A_0 is $\sigma^2 + \frac{n(n+1)}{12} a^2$ and that of A_1 is $\sigma^2 + \frac{a^2}{2}$.

Now ordinarily a is small compared to σ. Hence, in this case, A_1 is a better estimate of the variance than A_0. The expected value of the normalized square of the second difference,

$$A_2 = \frac{1}{6(n-2)} \sum_{i=1}^{n-2} (x_i + x_{i+2} - 2x_{i+1})^2, \text{ is } \sigma^2 + \frac{1}{6(n-2)} \sum_{i=1}^{n-2} (a_i + a_{i+2} - 2a_{i+1})^2.$$

Hence, if the trend is linear, this expectation is σ^2. Were it not for the small number of rounds (about 10) and the fact that in practice a^2 is a small fraction of σ^2, A_2 would be a better recommendation for an estimate of σ^2.

In fact if the trend is a polynomial of degree k then an estimate using k+1 differences removes the trend entirely.

D. Reduction Trajectories

Immediately before the reduction trajectories are begun a first estimate C_1 of the ballistic coefficient C is made from the average mass for each group and the caliber of the projectile, modified if experience dictates, by a form factor as in equation (4).

A function of the speed of sound, a, appropriate for use in either (3) or (6), e.g., $1/a^2$, is computed by hand from the previously reduced temperatures using (7). Using this function together with the previously reduced $\rho(y)$ and the first estimate C_1, a machine (Eniac, Edvac, or Ordvac) computes a reduction trajectory using the Heun method to integrate (1).

The next step (see the block flow chart, fig. 1) is to make a second estimate C_2 such that the range x_ω obtainable with it will lie on the other side of the observed range for the group from the range just computed with the first estimate C_1. Then another reduction trajectory is computed to verify or improve this guess.

It has been found from experience that a sufficiently close bracketing of the range has been obtained whenever

$$(8) \qquad |C_1 - C_2| < .005\, C_2.$$

Thus if (8) is not satisfied a new estimate of C is made by linear interpolation (with respect to the ranges) between C_1 and C_2. With this new C another reduction trajectory is computed and if (8) is now satisfied with C_2 and the new estimate then we go on to another set of reductions for another pair of initial elevation and velocity values. If (8) is not satisfied the procedure of new estimations of C and computations of reduction trajectories is repeated until (8) is satisfied. The number of repetitions seldom exceeds three.

When the computation of a set of trajectories for all given combinations of muzzle velocities and elevations has been accomplished, a fitting of the C's finally used in the computations by a low order polynomial in ϕ and V_0 is done as follows. First a family of graphs of C vs. ϕ with V_0 as a parameter and a family of graphs of C vs. V_0 with ϕ as a parameter are plotted. An experienced person determines visually whether a reasonable fit by least squares can be made by a constant C or by a linear or by a quadratic function of ϕ or V_0. It usually occurs that ϕ appears to a higher power than V_0 in these fits; occasionally V_0 does not appear at all; furthermore, the power of ϕ usually does not exceed 2.

The fit is a weighted least squares fit, given by minimizing

$$\sum_{\phi, V_o} \left[\frac{\partial x_\omega}{\partial C} \quad \frac{C-\bar{C}}{P.E._{obs.}(x_\omega)} \right]^2 \quad (\phi, V_o)$$

where $C(\phi, V_o)$ is the C obtained from the reduction trajectories, $\bar{C}$, $\frac{\partial x_\omega}{\partial C}$ is the change in range per unit change in C obtained from the last few reduction trajectories in each group, and $P.E._{obs.}(x_\omega)$ is the observed probable error in range. The criteria for the degree of the fit is that no lower degree fit produces an x_ω within one observed probable error of the x_ω computed in the appropriate reduction trajectories. If it so happens that a quadratic fit is not obtainable (as may happen when high angle values are needed in the table) over the entire range of ϕ and V, the independent fits are made over different portions of the range of ϕ.

In addition to the quantity $\partial x_\omega/\partial C$ the quantities $\partial x_\omega/\partial V_o$ and $\partial x_\omega/\partial\phi$ are obtained from the last few reduction trajectories. They are used in a statistical formula for the probable error in range due to special effects. The details of this formula are too specific to be discussed here beyond remarking that it involves these derivatives and sample probable errors in muzzle speed, mass, elevation, and form factors. If these special effects contribute substantially to the observed probable error, a new jump correction is made to reduce the contribution to these effects and a new fit to C is made. Another use of these quantities is made later in obtaining the constants in a fitted formula for the observed probable errors in range and deflection.

E. Normal Trajectories and Differential Effects

Once the proper ballistic coefficient is found, trajectories are computed under standard conditions for each velocity zone and a set of angles of elevation. These standard conditions are that there be no wind ($w_x = w_z = 0$), no rotation of earth effect ($\lambda_1 = \lambda_2 = \lambda_3 = 0$), no variation in g, and a standard atmosphere. At present a standard atmosphere is taken as one for which

$$\rho = \rho_s e^{-hy}, \quad a = a_s e^{-Ay}, \quad h = \frac{21}{2} A, \tag{9}$$

where A is a constant. As a result of these conditions, not only do the first equations of (1) simplify, but also the third becomes unnecessary since the trajectories would lie in a plane, there being no sidewise forces and, hence, no deflection.

These normal trajectories give range and time of flight for each muzzle speed and elevation. For each muzzle speed interpolations are performed on the set of ranges vs elevations to give range as the independent variable in 100 yard intervals with elevation, time of flight and components of terminal velocity as the dependent variable.

From this interpolation is obtained immediately the differential effect, change in elevation for 100 yard change in range.

Further differential effects are obtained by computing a few additional normal trajectories with changes in air density, temperature, range wind and muzzle speed. These give additional items for the tables and for later use in computing probable errors to be entered in the tables and to be used in computing fork.

During the course of the computations of each normal trajectory the values of x, y, $\dot{x}$, and $\dot{y}$ as functions of t are printed out. This is to be used later in obtaining table entries for angle of site corrections to the gun elevation.

One also gets from the normal trajectories the values of terminal speed and position of summit.

F. Firing Table Elements

The firing tables are arranged in sections with a fixed muzzle velocity in each section. Within such a section the independent variable is the range, given in hundred yard intervals. We have already mentioned how the corresponding elevations and variations in range with muzzle velocity, temperature, wind and density are obtained from the effects trajectories. All these quantities are also tabulated. Among the most important table elements remaining are corrected time of flight, drift, probable errors in range and deflection, fork, angle of fall, and complementary angle of site for each mil of site.

The corrected time of flight is obtained by adding to the computed time of flight a correction Δt. The formula for this correction is gotten by a least squares fitting of linear form in ϕ and V_o (or some other simple function of these which increases more rapidly when ϕ exceeds 65^o) to the differences between the observed times of flight and the times of flight of the reduction trajectories.

The drift is the portion of the deflection which is not due to wind or rotation of earth and is attributed to the spin of the projectile. The values of drift in the table are obtained from a formula which is a least squares fit to the drifts in the form

$$D = d\, t_{\omega}^{\,n}$$

or in the form*

$$D = k \tan \phi$$

where k and n are fitted constants, n usually being near 2.

The effect on deflection due to a unit cross wind is reasonably approximated by the formula

$$W.D. = \frac{1.01}{x_\omega} \, w_z \left(t_\omega - \frac{x_\omega}{x_o}\right)$$

where W. D. is wind deflection angle.

The formulas for probable errors in range and deflection are now obtained. The probable error in deflection is given by

$$P.\,E._D = \frac{x_\omega \tan \beta}{\cos \phi}$$

where

$$\beta = \underset{\phi}{\mathrm{av}} \left\{ \tan^{-1} \frac{(P.E._D)_{obs.} \cos \phi}{x_\omega} \right\}$$

where $(P.E._D)_{obs.}$ is the weighted observed $P.E._D$ for each ϕ. The probable errors in range are usually given by a least squares fitting

$$(P.E._{x_\omega})^2 = (P.E._{V_o})^2 \left(\frac{\Delta x_\omega}{\Delta V_o}\right)^2 + (P.E._m)^2 \left(\frac{\Delta x_\omega}{\Delta C}\right) \frac{C}{m} \times$$

$$\left[\left(\frac{\Delta x_\omega}{\Delta C}\right) \frac{C}{m} + 2 \left(\frac{\Delta x_\omega}{\Delta C}\right) \frac{nV_o}{m}\right] + (P.E._\phi)^2 \left(\frac{\Delta x_\omega}{\Delta \phi}\right)^2 + (P.E._C)^2 \left(\frac{\Delta x_\omega}{\Delta C}\right)^2 ,$$

---

* There appears to be some physical basis for the first formula above, but none for the second. Recently the Firing Table Branch of the Computing Laboratory has been trying to fit a drift formula (4.14 or 4.15 in 5) dependent on the speed along the trajectory, the spin, some physical constants of the shell and some aerodynamics coefficients dependent on Mach number.

where $\frac{\Delta x_\omega}{\Delta V_o}$, $\frac{\Delta x_\omega}{\Delta C}$ $(= -\frac{\Delta x_\omega}{\Delta \rho})$, and $\frac{\Delta x_\omega}{\Delta \phi}$ are obtained from the effects trajectories, and some of the probable errors on the right hand side are either observed or are obtained from the reduction trajectories and the others are fitted constants.

The range probable error besides being a table element is also used to obtain fork, the change in elevation (in mils) for a change of four probable errors in range; fork is given by

$$F = \frac{4c\ (P.E.)}{100} x_\omega$$

where c is the change in elevation for 100 yards change in range.

The angle of fall is obtained from the inverse tangent of the ratio of the vertical and horizontal components of terminal velocity obtained in the computation of the normal trajectories.

The complementary angle of site entries in the table are obtained either by hand or more recently on the ENIAC from two interpolations on the printed values of x, and y in the normal trajectory computations. The first interpolation is to get x as the independent variable in intervals of 100 yards. The second interpolation is on the values of y for each x to obtain ϕ from which one gets immediately the angle of site corrections.

These table elements, those mentioned in the previous section, and a few others not mentioned here are then subtabulated and rearranged for final tabulation and typing before the tables are reproduced and bound.

IV - BOMBING TABLES

A. General Remarks

Bombing tables are bomb aiming data used by bombardiers, bomb and sight manufacturers and others. Paralleling the case of firing tables, these tables are produced by

a) making of a series of bomb drops on the range and measuring whatever is necessary including non-standard conditions at the time of range drop,

b) computing reduction trajectories to obtain a drag coefficient K_D and a ballistic coefficient characteristic of the particular bomb,

c) computing a series of normal trajectories with the chosen K_D and ballistic coefficient, and

d) computing various correction formulas and probable errors for use in corrections for non-standard conditions occurring at time of drop.

A block flow chart describing these steps in further detail appears in Figure 2.

We now describe what is done in the main blocks of this flow chart to obtain finally the bombing table.

B. Collection of Range Data

At the bombing range groups of about 10 bomb drops are made for each of about 2 air speeds and for each of about 5 or 6 altitudes of release. Most drops are made from airplanes in horizontal flight. The speed of the airplane at bomb release and the release coordinates are obtained from ballistic cameras or from cameras obscuras, the choice depending on the bombing range at which the drops are made. These are synchronized with a chronograph from which is obtained the release time. The bomb is tracked in flight by means of Askania cameras also synchronized with the chronograph. The Askania films furnish azimuth and elevation of the bomb vs. time; supplementary information as a check on release time and coordinates are also obtained from the Askanias. The time of impact is obtained from geophones synchronized with the chronograph. From Mitchell or Bowen-Knapp cameras, also synchronized with the chronograph, data for striking velocity and supplementary data for time of flight are available. The impact point is obtained from theodolite data.

While the drops are going on several meterological data-gathering runs are made. The data obtained are the wind components down range and across range for equal intervals in time, at altitudes up to the release altitudes, and the density, temperature, and humidity at unequal intervals in altitude (equal intervals in pressure). The data are obtained as on the firing ranges, using instruments in balloons whose ascensions and drifts are tracked by theodolites.

These raw data must then be processed by preliminary computations, film readings, and reductions to obtain functional and numerical parameters appearing in the reduction trajectories. The purpose of the reduction trajectory computations here is not merely to obtain a ballistic coefficient as in the case of firing tables but also to obtain a drag coefficient $K_D(M)$.

C. Preliminary Reductions

The numerical parameters are the mass of the bomb and initial conditions required for solving the trajectories (1), namely, release coordinates, x_o, y_o, z_o, release speed, v_o, and angle of release (usually close to horizontal).

The functional parameters required are the drag coefficient $K_D(M)$, the air density $\rho(y)$, the speed of sound $a(y)$, the absolute temperature $T(y)$, and the wind components $w_x(y)$, $w_z(y)$. The speed of sound, $a(y)$, is obtained from $T(y)$ as in equation (7). The density $\rho(y)$ is obtained from $T(y)$ and the humidity.

The meteorological data obtained are reduced and interpolated to give air density, temperature, and range winds and crosswinds aloft at equal intervals (500 ft.) in altitude.

The various films of the release, trajectory, and impact are read (on Askania film readers, Telereaders, or Iconologs), the resultant data being prepared for triangulation and interpolation on IBM machines to produce on ORDVAC position, velocity, and acceleration of the bomb at various time intervals from release to impact. The IBM CPC merely adds tracking corrections to the readings and also takes differences from these results to check for film reading errors.

The positions, velocities, and accelerations obtained are then used with the metro data in the trajectory equations to get $K_D(M)$ at various points along each trajectory from the equation

$$(10) \qquad K_D(M) = \frac{m}{d^2 \rho(y) u^2} \sqrt{\ddot{x}^2 + (\ddot{y} + g)^2 + \ddot{z}^2},$$

where d is the diameter of the bomb and m is its mass. The rotation of earth and variation in g are neglected in (10) because (10) is to be used to get first estimates of $K_D(M)$ to be used in the reduction trajectories, and the effects of these phenomena are secondary.

The values of $K_D(M)$ obtained from (10) for each drop are then plotted against M, being plotted on the same paper for all the drops. Then a curve is drawn so that it passes close to the averages of these points. The values obtained from this curve are then the values of the first approximation to $K_D(M)$ to be used in the reduction trajectories.

D. Reduction Trajectories

With a choice of form factor, i, equal to 1 and the first approximation to $K_D(M)$ just described, several reduction trajectories from each group (with similar initial conditions) are computed. These trajectories use the air structure observed at the time nearest the time of the drop to obtain the wind forces on the bomb and the speed of sound $a(y)$; they also use the variable g given by equation (2).

Next, for each drop, one computes at the same altitudes along the trajectories the difference between the corresponding computed and observed ranges and times. If all these "residuals" do not have constant signs along the trajectory, the K_D curve is adjusted until all residuals of each type do have constant signs (usually opposite for range and time). Adjustments of the K_D curve for lower values of M are made using data from trajectories of low altitude drops, and for higher altitude drops. Then the form factor, i, is adjusted by either an increase or a decrease of 10%, depending on whether the computed ranges are over or short with respect to the observed ranges. By linear interpolation a new value of i isobtained which yields a better approximation to the observed ranges.

The computed ranges and times of impact for each group are compared with the observed ranges and times of impact by comparing the mean residual of the terminal values with the probable errors of these same residuals. The objective is to get each of these bracketed within the probable error for each group. If any group mean is greater than one probable error for the group, then either the K_D or the i is adjusted. The adjustment is usually made to reduce as much as possible the mean of the group whose mean had been the greatest in absolute value. This is accomplished by changing i, which is done by using the values of $\Delta x_\omega / \Delta i$ obtained from the earlier reduction trajectories. If most of the mean residuals in range to impact are of one sign and if the mean residuals in time of impact are of the other, adjustments in i are made. Otherwise, the K_D's are adjusted as before. When enough bombing range data are provided several repetitions of the above-described procedure suffice to attain the objective mentioned above.

The last value of i used is multiplied by the last K_D curve used and this resulting curve is defined to be the K_D for the bomb. Henceforth the form factor of the bomb is considered to be 1.

E. Normal Trajectories and Differential Effects

Once the proper K_D for the bomb is found, trajectories are computed under standard conditions for a large number of altitudes and speeds of release. The standard conditions are, as with the firing tables, a standard atmosphere as in (9), no wind, and no rotation of earth effects. However g is still permitted to vary with altitude, just as in the reduction trajectories.

Again there is no deflection, and hence no z-equation.

These normal trajectories give range and time of flight for each altitude and true air speed. Most usually there is no variation in the angle of drop, although requests for "climb and glide" information do appear from time to time. For use in computing the table elements the terminal values, x_ω, $\mathring{x}_\omega$, $\mathring{y}_\omega$, t_ω, are required as well as the quantities

$$\text{range lag: } B = v_0 \sqrt{\frac{2}{g}} \sqrt{y_0 - y_\omega} - x_\omega,$$

$$\text{time lag: } A = t_\omega - \sqrt{\frac{2}{g}} \sqrt{y_0 - y_\omega}.$$

Differential effects due to wind and height of target above sea level are obtainable from normal trajectories. Information for the differential effects due to height of target are obtained during computation of the trajectory by printing the values for a height of 5000 feet. The differential effects due to wind are obtained from a five-point differentiation formula on x_ω vs. v_0 to get $\partial X/\partial V$. How this is used to get the wind effects will be explained later in the section on table elements.

F. Bombing Table Elements

A bombing table contains tabulations of

disk speed setting (DS),
trail setting in mils and feet,
time of flight setting in seconds,
and their differential corrections

all as variables dependent on

true air speed in knots at 10 knot intervals,
release altitude above target in feet at, say, 100 foot intervals at low altitudes (e.g., below 12,000 feet) and at, say, 1000 foot intervals at high altitudes,
and the ratio, q, of differential ballistic wind to wind at bombing altitude, at intervals of 0.1 from 0.0 to 0.7. ($q = 0.1$ is omitted).

The differential corrections are due to the effects of range wind and height of target above sea level. Also included is a differential correction, due to cross-wind effects, called the aiming point offset upwind in feet for a 10 degree drift. There is also an entry incidental as far as bombing is concerned called dropping angle and its differential corrections, given in degrees, as functions of ground speed and release altitude.

At present, the ENIAC obtains the bomb sight settings by the following procedure:

(a) Use a 5 point interpolation on the values of x_ω and t_ω obtained in the computation of the normal trajectories to get them at 1000 foot intervals of release altitude.

(b) Use a 5 point interpolation on the values of x_ω and t_ω obtained in (a) to get them at regular intervals of release speed.

(c) Compute and subtabulate values of $k = t_\omega - \frac{\partial x_\omega}{\partial v_o}$ for various regular intervals in release altitude and release speed and the two heights of target, $y_\omega = o$ and $y_\omega = 5000$ feet ($\frac{\partial x_\omega}{\partial v_o}$ is obtained using a five point formula approximating the derivative.)

(d) Smooth (11 point 4th order formula) t_ω and k with respect to v_o to obtain them at the release altitudes, release speeds, and heights of target in (c).

(e) Smooth (as in (d)) t_ω and k obtained in (d) with respect to release altitude.

(f) Compute and put on cards

1. $t'_\omega = t_\omega - k_q$, (secs.)

2. $DS' = 5300/t_\omega'$, (disk speed)

3. $\lambda' = v_o t'_\omega - x_\omega$ (ft.),

4. $\mu = 1000/y^*$ (mils), where $y^* = y_o - y_\omega$,

5. $DS_F = \dfrac{5300\ v_o}{x_\omega + .23y^*}$ (whenever $\mu' > 230$ mils),

6. A.P.O. = $.17365\ \lambda'$ (ft.) (Aiming Point, Offset),

7. $DA_1 = \tan^{-1} x_\omega/y^*$ (dropping angle),

8. DA_2, $DA_3 = \tan^{-1} \dfrac{x_\omega \pm w_x t'_\omega}{y^*}$, $w_x = 168.894$ ft/sec,

9. x_ω,

10. $DS_{FW} = \dfrac{5300(v_o + w_x)}{x_\omega + .23\ y^* + w_x t'_\omega}$ (where $w_x = \pm 20, 40, 60, 80,$

100 knots in ft/sec, whenever $\mu' > 230$ mils).

(g) For bombing table elements below 12,000 feet from values of time lag A and range lag B obtained earlier interpolations as in (a) are performed to obtain A and B in 100 foot intervals in altitude; these are reconverted to time of flight and range from the range lag and time lag equations given above.

The IBM section, at present, then performs the necessary subtraction and retabulations to get data in form for printing on the electromatic typewriter.

G. Probable Errors

Although they do not appear in the bombing tables, probable errors in range and in time are desired for the general report on each program of range bombing and of subsequent production of bombing tables. There are two types of probable errors computed.

One set of probable errors in range and in time is obtained from the computation of the last reduction trajectories, with the accepted K_D for the bomb, and is used as a measure of the goodness of the fit of the K_D. The probable errors are computed for each altitude and speed group. They are obtained from the sum of the squared deviations of the residuals (range or time) from the mean deviation.

The other set is a set of probable errors in range and in time for each group of bomb drops reduced to standard conditions and, hence, is obtained from extra computations of normal trajectories. These probable errors are used as a measure of the goodness of the performance of the bomb. They are computed as follows. From the residuals used in computing the first set of probable errors, and from the quantity $\Delta x_\omega / \Delta i$ obtained in the reduction trajectory program, two sets (one for range and the other for time) of values of i are obtained. These values of i are chosen so that if the trajectories had been computed with them instead of with i = 1 the residuals would have vanished. Now extra normal trajectories are run with values of i bracketing the above to obtain ranges and times. By interpolation (linear) ranges and times corresponding to the above-mentioned values of i are computed. The probable errors of the residuals of these ranges and times for each group are then computed.

V - CONCLUDING REMARKS

A. Comparison of Computational Procedures

It was remarked at the end of the introduction that the block flow charts (Figures 1 and 2) of the firing and bombing table computational procedures reveal basic likenesses and differences. The mass of detail implied in each block of the charts described in Sections III and IV somewhat obscures these comparisons. It is of interest to review them.

The likenesses appear in the structure. The main blocks of the charts are the same: data are gathered, preliminary reductions are made, reduction trajectories are computed to reduce to standard conditions, normal trajectories are computed at standard conditions together with differential effects, probable errors are computed, and then the tables are produced. The chief causes of the likenesses are that the physical model and the numerical procedure employed are essentially the same for both. These were described in Section II. Finally both produce terminal values under standard conditions toward the end of the computations (i.e. from the normal trajectory).

The differences appear within the main blocks and have two main causes. These are the measurement methods on the one hand, and the different kinematic conditions caused by the different tactical requirements on the other.

As has been briefly remarked in the Introduction, in ground to ground range firings, the measuring equipment being used does not track the missile. Thus only initial and terminal information is available. Hence, the firing table computation up through reduction trajectories is forced to be a group and statistical reduction controlled by terminal conditions and producing a ballistic coefficient. The remainder of the firing table computations are directed toward obtaining tabulations of parameters and differential corrections useful to the man in the field who has to compute gun settings (i.e. the azimuth and elevation for firing). The tactics of artillery has been such that it has not been necessary to do these computations a few seconds before firing; they could have been done as much as an hour before. Furthermore the tactical problem is such that the whole trajectory is involved, but an accurate control of range is much more essential than an accurate control of time of flight. (This should not imply that an accurate knowledge of the time of flight is not desireable. It is needed in fusing).

On the other hand, in the air to ground problem of bombing from aircraft the measuring equipment tracks the missile. This permits a round by round reduction producing a K_D when the reduction trajectories are completed. However, in this problem only the down part of the trajectory is involved, but the time of flight accuracy is just as essential as the range accuracy. Also the tactics of bombing require that the computation of the release time be done within seconds preceding release; the computing of release time must, therefore, be automatic. Hence, the computing done after the normal trajectories are finished is directed toward tabulations of appropriate settings of a bomb sight, which automatically gets the release time.

These differences account for the different processing of data and the different timing and arrangement of personnel to do the work of producing the tables. The difference due to method of measurement can easily disappear. As an example, for rocket and missile computations the Firing Tables Branch does K_D reductions. The possibility of Doppler and accelerometer measurements has already been mentioned in the Introduction. However, the differences caused by tactical requirements can be much more essential. As an example, in ground to air firing (e.g. anti-aircraft), only the ascending portion of the trajectory is involved, and time of flight must be accurately computed, whereas range is of lesser importance, especially when proximity fuses are used. As a last example, the air to air firing tables are somewhat different. The time of flight is of utmost importance, range being a minor consideration. Due to the relatively high missile speeds and relatively short distances between gun and target, in this case only the flatter portions of the trajectories are computed. Thus, a Siacci ballistic theory (see, e. g., Bliss [1]) is used instead of the theory mentioned in Section II. In the computation of some of these tables much more than in other types of tables, the problem of the large yaws encountered by missiles fired at large angles to the direction of the plane's motion present substantial difficulties. This introduces another variation in the physical theory used (see, e.g., Sterne [7]). Finally, the table elements, slant range and drop needed here are not required in other types of tables.

B. Time Requirements

It is now of interest to have a comparative idea of the amount of time required in each of the blocks of figures 1 and 2.

We consider the firing tables first.

The range firings normally take from three to six weeks. The initial reductions at the range usually take two weeks or so, but overlap the firing time except for maybe a week over. The further reductions, before running reduction trajectories, consume roughly four man weeks for a program of thirty or so groups. Such a program would call for about one hundred and fifty reduction trajectories. While each such trajectory takes some fifteen minutes on the ENIAC (depending, of course, on the time of flight) and some two minutes on ORDVAC (when print-outs for every second of trajectory are not required), the total time for the reduction trajectories usually consumes from one to five weeks, depending on the priority. One to two weeks are then needed to fit for ballistic coefficients. Some three thousand normal and effects trajectories are then needed, computed at the rate of about 80 to 100 per hour on ORDVAC and about 30 per hour on ENIAC. It is usual to do these computations over a period of about six to eight weeks. During this time probable errors and incidental fittings are computed. The production of table elements then requires some four weeks. Finally eight to ten weeks are needed to transcribe the table elements. This completes the work done by the Computing Laboratory.

The bombing table running times are as follows.

The range bombings and appropriate data gathering occupy anywhere from six weeks to a year and a half depending on the size and urgency of the program, with many taking about half a year. A small program will involve about thirty drops where large ones have around two hundred. It takes about one half a man day to do one reduction of a ballistic camera plate, and another half man day for metro data. Two to three months are then needed to read the Askania films and convert the readings to the form from which the initial estimate of K_D is made. This conversion, described by (10), takes about ten to fifteen seconds per point on ORDVAC. There are about six to twelve of these points per drop. This amounts to a total time of about four hours or so of computing. These computations are done at scattered times while the Askania readings are going on. The reduction trajectories, of which there are usually around 400 to 600, take less than four days, about fifteen being done per hour on ORDVAC and about eight to ten per hour on ENIAC with printouts every second of trajectory. Around 600 normal trajectories are needed. They are computed at about 100 to 130 per hour on ORDVAC and about 30 or so an hour on ENIAC. The ORDVAC computations are usually spread out over about two to three shifts while on ENIAC they are spread over about three shifts. The probable errors take about four man days of hand computing to do about 150 of them and they can be done while the table elements are being transcribed. The table elements are computed on ENIAC in about fifty to sixty hours. The transcription of the bombing tables is usually accomplished with about one day of rearrangements on IBM followed by about two weeks on the electromatic typewriter. When this is done by hand it takes about one and one-half months or more of typing. This completes the work of the Computing Laboratory.

C. Improvements

We conclude now with a sketch of improvements recently introduced or under active consideration and further suggestions for possible improvement in the computational procedures. Unless statement is made to the contrary, all suggestions below were either initiated by the authors or by coders within the Analysis and Computation Branch of the Computing Laboratory.

First of all, T. E. Sterne suggested in August 1952 that the process of estimating a first K_D in bomb reductions be simplified by not using every single frame of an Askania film record of a drop. By choosing an unequally spaced set of frames to smooth for position, velocity, and acceleration at the mid-point, one can not only obtain sufficient K_D information for the drop, but one can also see to it that the results at different points are independent of one another, thereby eliminating possible fictitious oscillations in the reduced data. The choice suggested reduces the number of Askania readings to approximately one sixth of what it was.

Secondly, the physical model described in Section II is being modified by Mr. H. Reed of the Computing Laboratory's Firing Table Branch to a particle trajectory theory that includes a lift force as well as a drag force. This is being tried with the hope that in the case of high angle trajectories not only will the already achieved matching to time of flight and range be obtained, but a better fit to actual height of summit will result.

Thirdly, whereas hitherto metro data for bombing have been interpolated to equally spaced altitude intervals as a separate computing operation before the reduction trajectories are begun, the reduction trajectory computation is now being remechanized to work directly from the unreduced metro data.

Fourthly, another mechanization in progress is concerned with the transcription of table elements. Up to now the table elements printed out by the tabulators attached to the ENIAC and the CPC have been recopied in the desired format by a typist, taking about eight to ten weeks. At present this work is being mechanized in such a manner that the table elements are so arranged on the output cards that the proper format can be printed out directly on the electromatic typewriter in a few weeks. (All that is left is the additional typing of the headings, introduction, and a few small hand computed tabulations.) At present this mechanization is being developed on the ENIAC and can possibly be done on the CPC.

Fifthly, in connection with the metro data, we have noted the somewhat more complicated metro reduction required by firing table computation because of the fact that several metro runs may occur during a single group of firings. It is suggested that nevertheless this reduction can be mechanized without much difficulty. Also there should be no difficulty involved in mechanizing the transformation of metro temperature data to a/a_s.

Sixthly, it has been suggested that two input card readers instead of one could make the ENIAC and the IBM-CPC much more efficient and rapid in performing interpolation in two variables. If this extra equipment were installed, the computation of the initial estimate of the K_D for bombs obtained from the metro data in equal altitude intervals and the Askania data in equal time intervals would be done much faster on the ENIAC and could even be done on the CPC.

Seventhly, W. W. Leutert has pointed out that with the introduction of the static magnetic memory on the ENIAC it is now possible to introduce further mechanizations of programs, already used on EDVAC and ORDVAC, onto that machine. Two examples are the variation of Δt (see end of Section II) and the bracketing (see beginning of Section III D) of the ballistic coefficient.

Eighthly, W. W. Leutert has further pointed out that the G functions in the firing table computations be approximated by analytic expressions for use on the machines. In this connection, see the last suggestion below.

Finally, there is the possibility of completely mechanizing the whole procedure of both running the reduction trajectories and determining of the final K_D to be used in the tables. This would seem to be feasible when large memories become available on the machines.

With the present memories of the BRL machines, a partial mechanization appears possible if memory requirements for the K_D are reduced. This would seem to require putting in either only a few points or an analytic representation (e.g., polynomial segments) for the K_D curve and possibly doing the same for the metro data. We might add that various suggestions and attempts to represent K_D and G by analytic expressions have been frequently made and for a long time. (See, e.g., a suggestion by Morrey [12]).

At present the following improvement suggested by W. W. Leutert is being attempted for bombing table reductions. Each K_D curve is to be approximated by several low order polynomial segments using least squares fitting. The coefficients of these polynomials (instead of the much larger number of tabular values of the K_D curve) are then to be put into the memory of the machine. This permits a program for the determination of i (See figure 2.), all the raw metro data, and a program for interpolation in the metro data to enter the memory at the same time. This will save the present separate metro reduction runs, the present repeated reading in of the metro data during the determination of i, and the punching of the corresponding K_D's. It is estimated roughly that such a procedure will save an hour or so of metro reduction runs and about one-third to one-half the machine time now spent on the reduction trajectory program. This is achieved at a cost of no more than about two man-days of least square fittings for each K_D, most of which is taken up by the preparation of input data. A further saving is in the considerable reduction of the possibilities of errors in punching of input data, which can lead frequently to wasted machine time. This saving is difficult to measure but would appear not to be negligible. Finally, such a method opens up the possibility of experimenting to see whether a sufficiently accurate K_D cannot be obtained from the data of relatively few bomb drops. The resulting saving would be considerable, since dropping bombs from aircraft is expensive.

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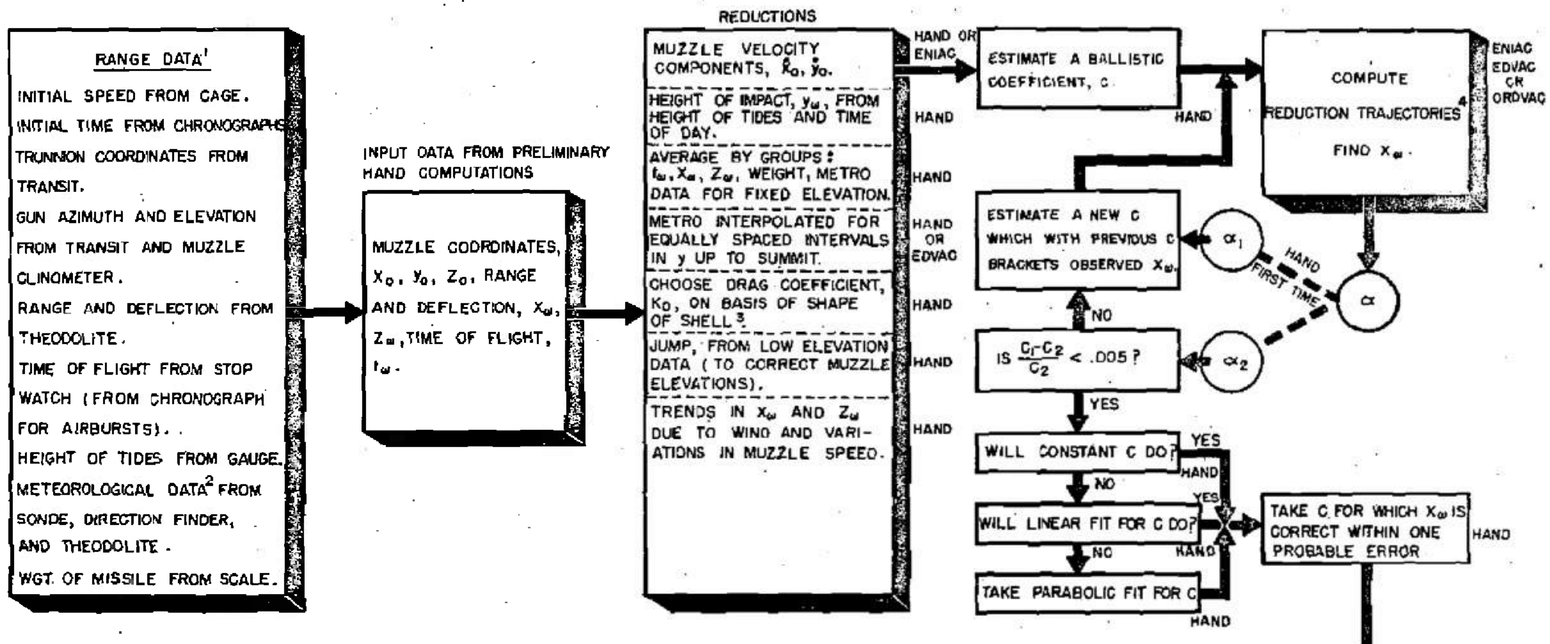
* In item 4, chapter 10 was written by R. H. Kent and L. S. Dederick,
chapter 11 was written by R. H. Kent, and
chapter 12 was written by Col. H. H. Zornig and R. H. Kent

BLOCK FLOW CHART FOR COMPUTATION OF FIRING TABLES

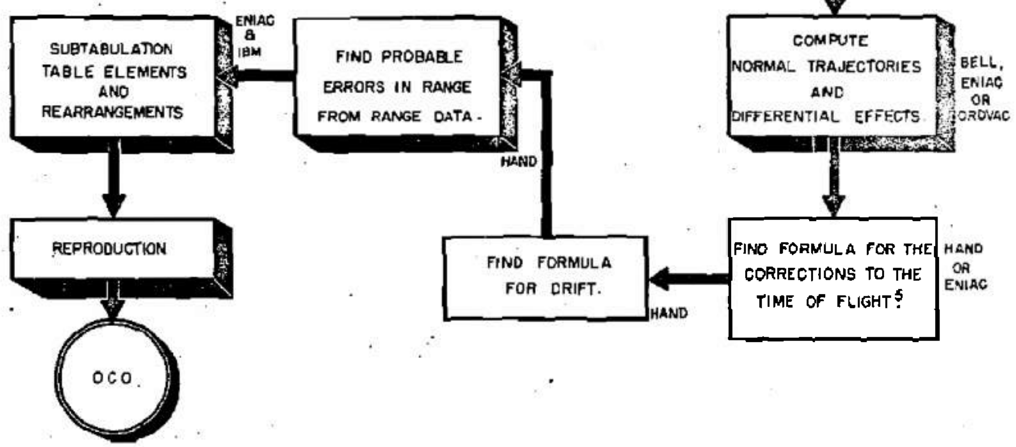


NOTES:

1. DATA GATHERED FOR APPROXIMATELY 10 RD. GROUPS, 5 OR 6 GROUPS PER VELOCITY ZONE, 1 TO 8 ZONES.
2. METEOROLOGICAL DATA INCLUDE RANGE WIND W_x, CROSS WIND W_z, AIR DENSITY ρ, AIR TEMPERATURE, AND HUMIDITY.
3. SOMETIMES A DRAG FUNCTION, $G(V^2/100)$ IS CHOSEN INSTEAD OF CHOOSING A DRAG COEFFICIENT.
4. FINAL REDUCTION TRAJECTORY IS COMPUTED USING LAST COMPUTED C FOR EACH GROUP. LAST FEW REDUCTION TRAJECTORIES YIELD SUCH SPECIAL EFFECTS AS $\frac{\partial X_\omega}{\partial C}$, $\frac{\partial X_\omega}{\partial V_0}$, $\frac{\partial X_\omega}{\partial \phi}$, AND PARTS OF z DUE TO ROTATION OF EARTH AND CROSSWIND.
5. THE t_ω CORRECTION FORMULA IS FOUND BEFORE THE LAST NORMAL TRAJECTORIES ARE COMPUTED.

ADDITIONAL NOTE:

THE BOXES WITH SHADOW CONTAIN SIMILAR OPERATIONS TO THOSE IN BOXES WITH SHADOW IN BOMBING TABLE FLOW CHART.



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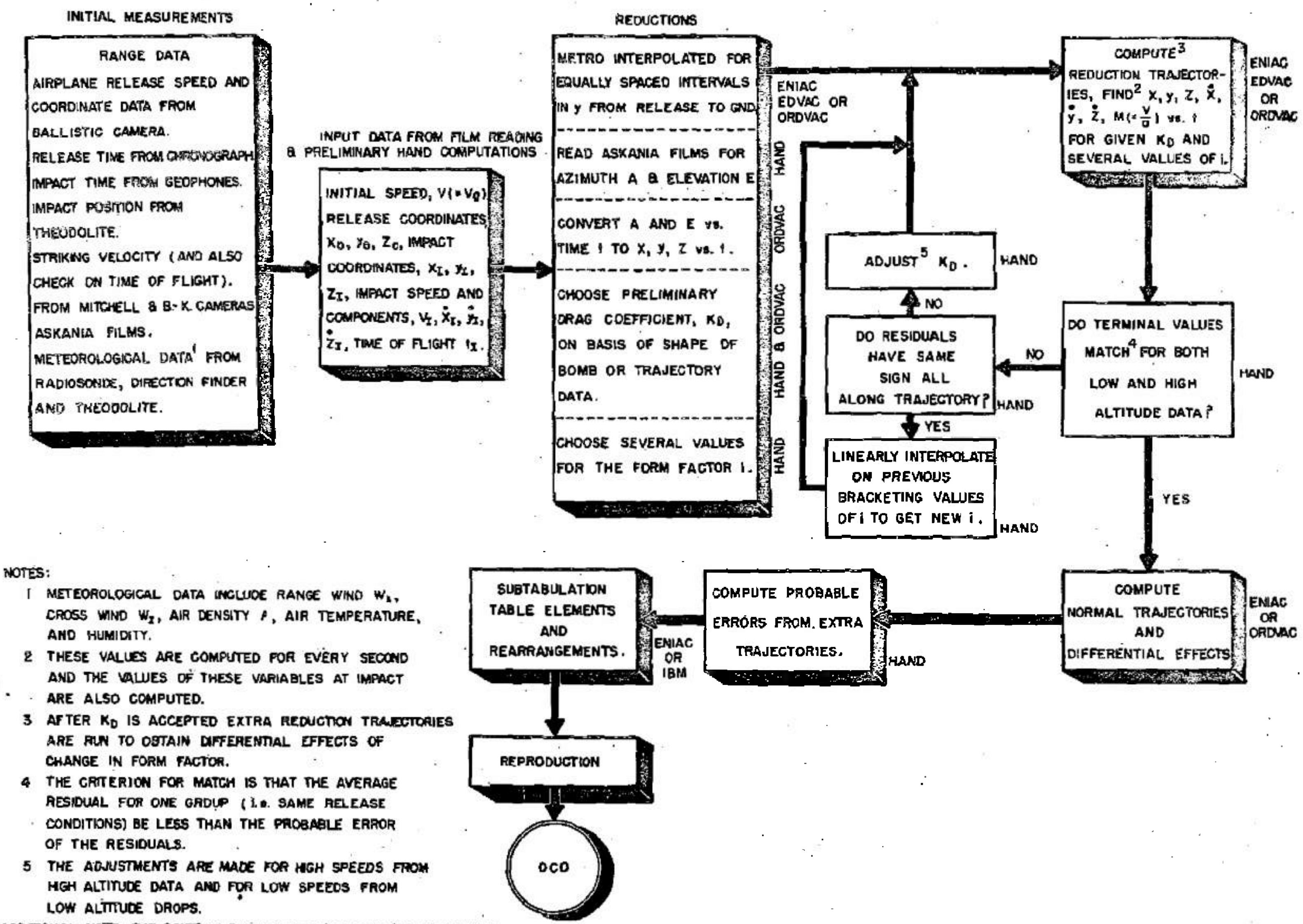
BLOCK FLOW CHART FOR COMPUTATION OF BOMBING TABLES



NOTES:

1. METEOROLOGICAL DATA INCLUDE RANGE WIND W_x, CROSS WIND W_z, AIR DENSITY ρ, AIR TEMPERATURE, AND HUMIDITY.
2. THESE VALUES ARE COMPUTED FOR EVERY SECOND AND THE VALUES OF THESE VARIABLES AT IMPACT ARE ALSO COMPUTED.
3. AFTER K_D IS ACCEPTED EXTRA REDUCTION TRAJECTORIES ARE RUN TO OBTAIN DIFFERENTIAL EFFECTS OF CHANGE IN FORM FACTOR.
4. THE CRITERION FOR MATCH IS THAT THE AVERAGE RESIDUAL FOR ONE GROUP (i.e. SAME RELEASE CONDITIONS) BE LESS THAN THE PROBABLE ERROR OF THE RESIDUALS.
5. THE ADJUSTMENTS ARE MADE FOR HIGH SPEEDS FROM HIGH ALTITUDE DATA AND FOR LOW SPEEDS FROM LOW ALTITUDE DROPS.

ADDITIONAL NOTE: THE BOXES IN SHADOW CONTAIN SIMILAR OPERATIONS TO THOSE IN BOXES WITH SHADOW IN FIRING TABLE FLOW CHART.

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